SCAFFOLDING STUDENTS TO UNPACK A MODELLING PROBLEM

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In this session, I will discuss one way to scaffold the development of students' modelling capabilities – through the use of a handbook structured according to key phases of the modelling process.

The session will have a particular emphasis on how to assist students in developing the relevant mathematical question and related assumptions that will direct their efforts to generate a solution.

The session will provide opportunity to discuss the specific challenges associated with implementing modelling in the classroom.

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PROBLEM 8

Sausage Sizzle

The Fundraising for schools, fetes, and clubs often involves running stalls where sufficient items must be sold at a big enough profit to make the effort worthwhile. How do we know how much to charge to make this likely?

Problem: A school intends to organise a sausage sizzle at a fete. What price should be charged per sausage? – including bread and sauce

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Saving Las Arenas

Bullfighting was once an important cultural activity throughout Spain. Its popularity, however, has declined over time to the point where it is now banned in many parts of the country. This has resulted in bullrings that can no longer serve the purpose for which they were built.

One such instance is Las Arenas in Barcelona – which was, and still is, an important architectural landmark in this city. This bullring was closed in 1977 and remained largely unused until 1999, when it was purchased for redevelopment into a shopping centre. The project was completed in 2011 when Las Arenas was opened under its new guise (see for more detail https://barcelonalowdown.com/las-arenas-bullring-shopping-centre-panoramic-viewpoint/). An image of the redeveloped Las Arenas appears in Figure 1.

Figure 1: Las Arenas after development

Figure 2: Excavation of Las Arenas

The engineering challenges of refurbishing and old bullring into a shopping centre were huge. There were many aspects to its redevelopment including the construction of a roof, shop floor spaces and viewing tower. The focus of this task, however, is on the new foundations that were needed to support the "new" Las Arenas.

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You might imagine the complexity of costing of this project. For the purpose of this task, however, you are being asked to find the costs of the concrete foundations alone. Please develop your response on the basis of material costs in 2019.

The following information might be helpful:

Floors 5	**E-forum** 5,500m²	**Building Diameter** 100 m
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Please make use of whatever information you can find about Las Arenas on the internet or elsewhere to complete this task. The websites provided are good starting points.

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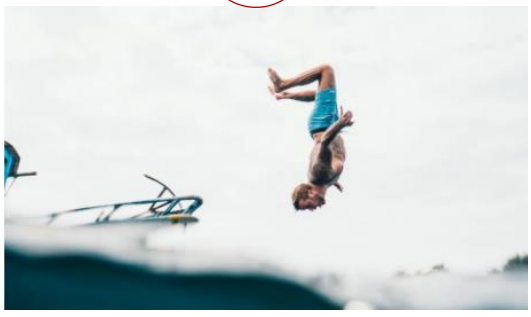
Diving in head-first

Why read the task?

Why not just ask your friend?

It is easy – the answer is $2.



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Cambridge text Year 8

6 A restaurant bill of $100 is to be paid. Blake puts in one-third of the amount in his wallet, leaving $60 to be paid by the other people at the table.

a Write an equation to describe this situation, if b represents the amount in Blake's wallet before he pays.

b Solve the equation algebraically, and hence state how much money Blake has in his wallet.

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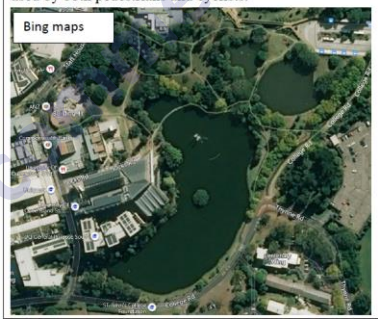
Context

Green spaces and small artificial lakes and ponds are important measures for flood control and enhancing the liveability of a city. Below are images of the University of Queensland lakes. These provide habitat for birds and other wildlife. The paths around the edges are peaceful walking tracks used by both pedestrians and cyclists.



Task

Design an artificial lake or pond that fits within the parkland of Robelle Domain Parklands (represented as green space in the screen shot below). You will need to decide what portion of the parkland will be used. Construct a real-life model of the boundary of your lake or pond using functions and relations. The boundary is to be a birds-eye or plan view of the lake in two dimensions. Sharp points and angles where water becomes stagnant and algae can bloom should be avoided.

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Mathematical Modelling problems

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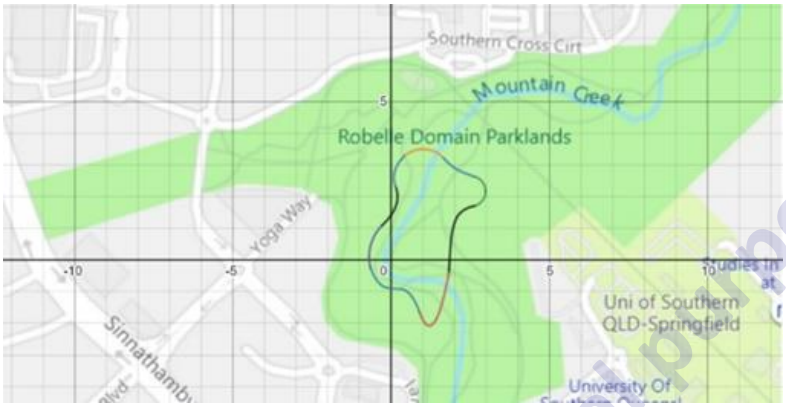

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Scaffolding through the process

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The key phases of mathematical modelling

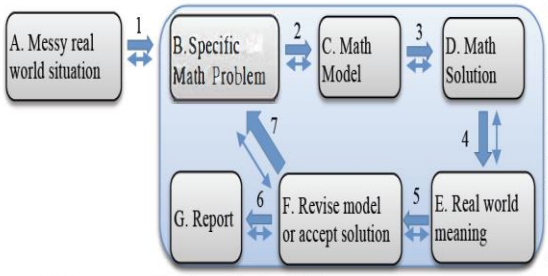

Figure 1. The process of mathematical modelling

1. Understanding, structure, simplifying, Interpreting context
2. Assuming, formulating, mathematising
3. Working mathematically
4. Interpreting mathematical output
5. Comparing, critiquing, validating
6. Communicating, justifying (if model is deemed satisfactory)
7. Revisiting the modelling process (if model is deemed unsatisfactory

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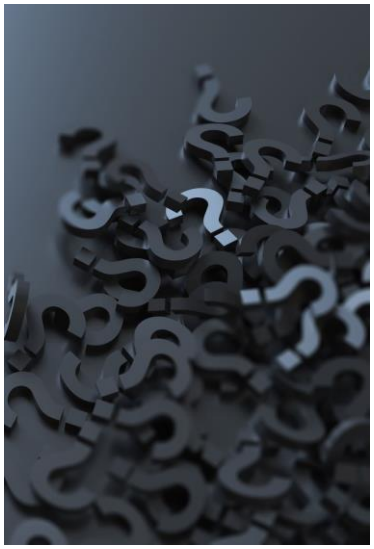
Modelling Process

Points that may be considered by teachers and students:

- It is necessary to leave the realm of pure mathematics to build a model, e.g., by procuring extra-mathematical information and data.
- Several different models may be reasonable. There is rarely a unique, or a best, answer to a modelling problem.
- Modelling is not a five-minutes-to-get-an answer activity.
- Simplifications are likely to be needed and assumptions may be necessary to reduce the complexity of the extra-mathematical domain being modelled or to make the mathematics tractable.
- Assumptions can be made at any point in the cycle.
- Students should be encouraged to ask clarifying questions..



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Initial problem presentation

Points that may be considered by teachers and students:

- Teachers use facilitating questions that emerge from students' engagement in the task rather than clarify problem contexts or ask questions up front. Responses should align with the question "What should a modeller be asking himself/herself at this point in the modelling process?" (metacognitive connection)
- There should be a focus on student decision making – with no prior indications of what specific mathematical question should be identified, or what mathematical content will be useful in addressing the problem
- Students should be encouraged to pose explorative questions as to the nature of the endeavour as well.

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Booklet

Saving Las Arenas

9MM

Problem Solving and Modelling Task

(Designed to be completed in groups of 4)

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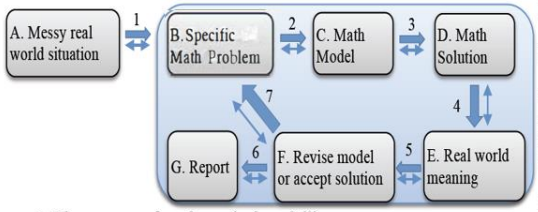

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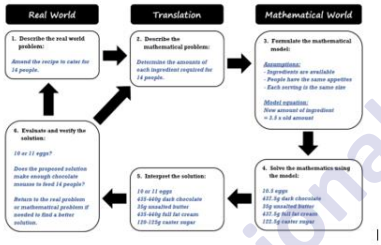
Example:



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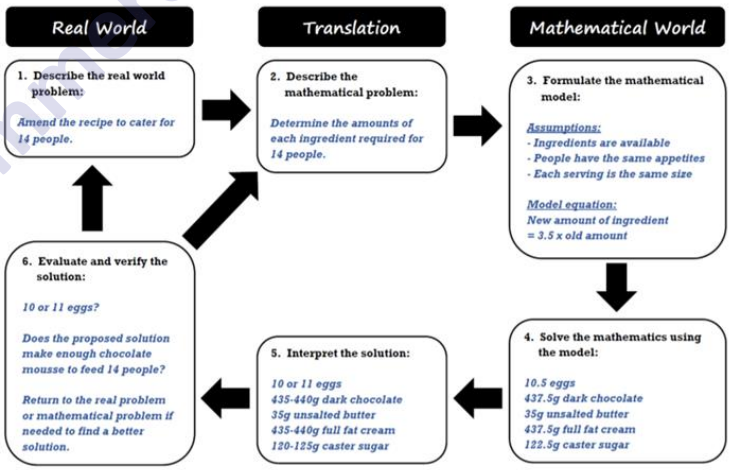
An example…



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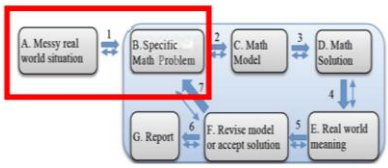
Page 3



Problem Statement – (Understanding, structure, simplifying, interpreting context)

[15 minutes (pages 3 and 4)– then one person from your group will share your mathematical question with the class.]

What is the messy real world problem?	What is the mathematical question? (Use mathematical vocabulary.)
Write a few sentences summarising the real world problem. Imagine you are explaining it to a friend who does not have the problem sheet.	

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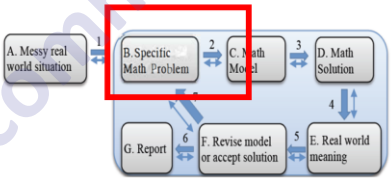
Page 4



Formulation – **(building the model- mathematical questions, assumptions)**

[30 minutes (pages 4 and 5) – then one person from your group will share your model with the class. Spend 10 minutes recording]

Thoughts, ideas and questions that define the problem.	**Assumptions:**

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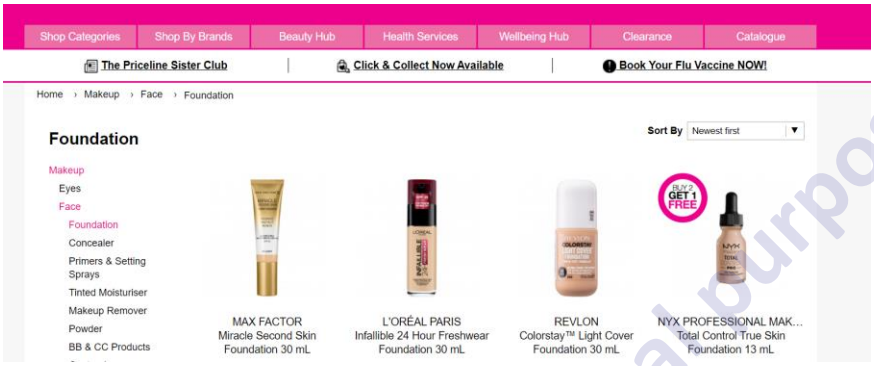
What is a foundation… Google searching.



https://www.priceline.com.au/cosmetics/face/foundation

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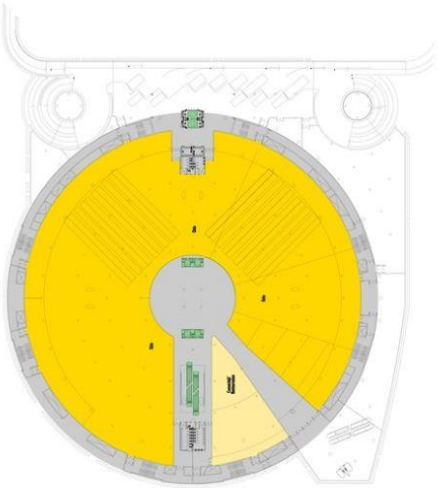

https://www.archdaily.com/530762/las-arenas-alonso-y-balaguer

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At the end of the booklet…

Report Structure:

Heading: Saving Las Arenas

Introduction

- **Describe the real world problem (page 3).**
- **Explain the mathematical question (page 3).**
- **Write a paragraph explaining how you defined the problem (page 4). You can also add other things you may have thought of during the task.**
- **Write a paragraph or dot points explaining the assumptions you made and why you made these assumptions (page 4).**
- **Explain any ideas or thoughts you had about the problem. Things you decided not to worry about as well as those you may have used to define the problem.**

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Sample of student report…

Intro: Las arenas was an old bullring renovated to become a shopping centre. We defined the problem as concrete cost but while defining the question we talked about the shape of the foundation and what shape it would be in which would then change the volume of concrete needed. We also questioned whether to include the cost of digging the foundation and paying for workers to lay the concrete increasing cost. Another thought was whether or not any mistakes could happen when building such as spillage or setting too quickly which would restart the progress and add to cost. We also thought about the type of concrete and whether it came in set blocks or as a pour. On top of this we questioned whether or not the concrete foundation had metal rods in it and if so if we had to pay for rods. When looking at blueprints we saw a carpark and wondered whether or not to include it as it would change foundation size. These all helped us define the question simply as how much for the concrete to go into the foundation.

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Sample of student report…

The assumptions we made were that we wouldn't pay for the labour or digging of hole and assumed no mistakes would be made while laying foundations out as the question purely asks about the concrete not how or who. We also assumed that the foundation was one large slab that outlined the area of the arena in a circle shape we presumed this as most foundations are in the general shape of the building they support. We assumed it was concrete set in blocks as it was a large area too large to be poured in as that would take days. We disregarded the carpark as it wasn't mentioned in the question or on the sheet. We were unsure what concrete was used so searched up average concrete price and used that. The last assumption made was whether or not we would pay for the metal rods in the foundation and if they were there or not but ruled it out as the sheet didn't mention it.

Anything that didn't appear on the sheet was ruled out as it was unknown which helped narrow down the question. I thought about the problem too

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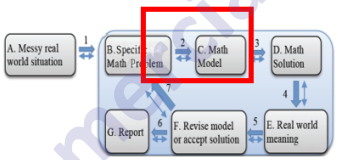

Formulation – **(building the model- choice of variables, parameters.)**

Models: A model can be an equation, a diagram or even a spreadsheet that we can use to solve a problem. **What is the best model to solve this problem?** **(If you think of more assumptions later add these on page 4)** **Citeace (record all relevant references used):**	**Parameters are the values that allow a model to define a particular situation. For example *m* and *c* in the equation of a straight line $y = mx+c$** **Define your variables:**

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A blank page was left for students to write their solution.

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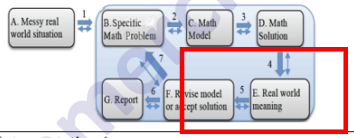

Interpretation (real world meaning of the solution)	**Evaluation** (compare, critique, validate)
What does the solution mean in the real world? **Does the solution require minor adjustments or do you need to change the original model in some way?**	**Does your answer make sense in the real world?** **Can you check your answers?** **Check all your working and assumptions.** **Could the model be made better? How would you do this?** **If you have time go ahead and try the better model, but make sure you write down the work you did before making the changes.**

Scaffolding through the process

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The Model

- **Define all variables (page 5). Remember to use "Let" statements.**
- **Explain the general model used (page 5)**
- **Explain why you decided this was the best model to use at this stage.**
  (If you changed the model later then explain the solution from the first model before showing how you changed the model)

Solution

- **Show full mathematical working using your best handwriting (page 6)**
- **Remember to line up the equal signs.**
- **Do not forget your units.**

Interpretation and Evaluation

- **Write your solution in terms of real world measurements (page 7). Remember we cannot have half an egg in the real world.**
- **Is your answer sensible, can you check or validate your answer using the information given?**
- **Write a paragraph explaining how you have checked your working and answer. You may decide at this stage that you need to adjust your model and find a better solution. Write how you made this decision and what changes you made. Explain why this would give a better solution.**

Conclusion – Write a paragraph summarising the task and the solution.

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