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Using mathematics to solve real world problems: The role of enablers

Design and Implementation Framework

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Foundation principles
The modelling process, including a graphical representation, should be understood.
The modelling process should be applied to a problem before it is presented to students – to be equipped to anticipate many of the contingencies.
Report documentation should develop in parallel with progress through the stages of the modelling process.

Principles for modelling task design	
Nature of problem	Problems must be open-ended and involve both intra- and extra- mathematical information. The degree of open-endedness is dependent on students' previous experience with modelling. Less experienced students may need additional scaffolding questions or information. More experienced students should be expected to engage with less defined problems.
Relevance and motivation	There is some genuine link with the real-world of the students. This will depend on factors including students' age, year level, personal circumstances, etc. Problems may need to be contextualised for specific student groups.
Accessibility	It is possible to identify and specify mathematically tractable questions from a general problem statement. Is there a mathematical approach accessible to students? Problems must be tractable from the perspective of the student group.
Feasibility of approach	Formulation of a solution process is feasible, involving (a) the use of mathematics available to students, (b) the making of necessary assumptions, and (c) the assembly of necessary data. Teachers must work through the problem.
Feasibility of outcome	Solution of the mathematics for a basic problem is possible for the students, together with interpretation. Expectations in relation to the type of response, for example, arithmetical versus generalised solutions, are dependent on the characteristics and year level of the specific student group being engaged.
Didactical flexibility	The problem may be structured into sequential questions that retain the integrity of the real situation. (Having worked through the problem, how can it be implemented?) For example, can prompts/assistance to students be structured into sequential questions (identify sub-sections of the problem)?

Pedagogical architecture		
Pre-engagement: Understanding of the modelling process and its application (learn/illustrate the modelling process)	Students need to be initially familiarised with the modelling process. This can be supported via materials including: • A copy of the of the modelling process (diagram/graphic) [modelling infrastructure] [also for students to map their way around the graphic during implementation]. • An example of a simple modelling problem matched to the phases of the cycle: problem statement; formulation; solution; interpretation; and evaluation • A copy of report structure. Students should have a clear idea what their report should look like at the end.	
Modelling process review	Reviewing pre-engagement as required. • The length of the discussion is dependent on students' prior experience with the modelling process. • Each student is provided with a copy of the modelling task, and a representation of the modelling process (e.g., a diagram) that is a depiction of the logical process that will guide their efforts.	Points that may be considered by teachers and students: • It is necessary to leave the realm of pure mathematics to build a model, e.g. by procuring extra-mathematical information and data. • Several different models may be reasonable. There is rarely a unique, or a best, answer to a modelling problem. • That modelling is not a five-minutes-to-get-an answer activity. • Simplifications are likely to be needed and assumptions may be necessary to reduce the complexity of the extra-mathematical domain being modelled or to make the mathematics tractable. • Assumptions can be made at any point in the cycle. • Students should be encouraged to ask clarifying questions.
Initial problem presentation	• Teacher provides brief general description of the task scenario [2-3 minutes]. • Students should be organised into small groups. They are provided with time to read the task description and identify a mathematical question(s) [5 minutes] • Teacher calls the class back together to discuss their initial understanding of the task and possible mathematical questions. Each group contributes via a representative. • Students in groups then consider assumptions and variables	Points that may be considered by teachers and students: • Teachers use facilitating questions that emerge from students' engagement in the task rather than clarify problem contexts or ask questions up front. Responses should align with the question "What should a modeller be asking himself/herself at this point in the modelling process?" (metacognitive connection) • There should be a focus on student decision making – with students required to initiate suggestions regarding relevant mathematical content, assumptions, variables; and for the more experienced, possible alternative questions. • Students should be encouraged to pose explorative questions as to the nature of the endeavour as well

	relevant to the mathematical question as well as other observations such as trends in data, etc. [5 minutes] • Teacher once more calls the class back together. Each group reports back to whole class by group representative. Teacher synthesizes/prioritizes students' initial assumptions and variables sufficient to begin modelling process for an initial model.	
Body of Lesson	Students: • Proceed in their groups to create model, solve, interpret, etc in terms of the question they are addressing. • Engage in productive student-student collaboration • Identify and make productive use of technology where applicable, for example, to source relevant information, check calculations and/or generate solutions. • Develop a report of their progress in terms of the stages of the modelling process (e.g., formulate, solve, interpret, evaluate) Teachers: • Bring to consciousness those things that are implicit …actions are then deliberate. • Activate teacher meta-meta cognition: (a) How will the students be interpreting what I as a teacher am doing/saying at this point? (b) What should the students be asking themselves at this point in the modelling process? • Support students with making progress through the modelling process. • Anticipate where students might have problems, e.g., interpreting the problem, generalizing the solution. • Employ measured responsiveness – rather than providing specific advice about the problem, teachers should prompt students to think about where they are in the modelling process. Structure mathematical questions that promote a viable solution pathway. • Encourage the use of digital or other tools as appropriate. • Support student development of a modelling report.	Points that may be considered by students: • Documenting progress against a visual representation of the modelling process. Problem statement → Formulate → Mathematical solutions → Interpreting outcomes → Evaluation. • Forms of collaboration: Working separately and then coming together; Working together from the beginning; Negotiating/confirming consensus; Explaining external to the group [Teacher/Researcher – Student). Students also encouraged to identify groups working on a similar problem/issue and extend collaborations. Points that may be considered by teachers: • Checking if documenting progress against the modelling process is taking place (both in the doing and in the recording). • Focusing students' attention on phases of the modelling cycle (there should be no specific direction towards a solution). • Support student decision making – multiple solution pathways should be encouraged. • Responses to students' questions or requests for assistance could include: What are you doing? What are you trying to do? Where are you in the modelling process? How have you checked your answer? (both mathematically and in terms of context); Can your solution be generalized? • Take account of student capability (catering for diversity)
Conclusion: Presentations of findings and teacher summary	• Students share what they have found with justification (representative from each group as spokesperson). Findings should be reported in a succinct fashion (e.g., via 3-4-minute video) • Teachers/students ask questions of clarification as required or to test arguments.	Points to be considered by teachers and students: • Students in the audience should provide commentary that includes questions, elaborations, clarifications (e.g., each student to write down one question or comment about the presented model). • Comments could also be directed towards criteria related to making judgements about the quality of the presentation of findings (e.g., Problem statement → Formulate → Mathematical solutions → Interpreting outcomes → Evaluation). All students should have access to these criteria. • Teacher clarification questions can include: How does that work with your model? (e.g., teacher has identified an error); Will your solution work for other situations? (e.g., teacher encouraging students to generalize). What did you do to evaluate the model? (e.g., teacher encouraging students to validate and verify am proposed solution). • The focus should be on what was learnt about the modelling process
Report (if required)	• Students should communicate their findings via a succinct, coherent, systematic report. The report must make use of appropriate mathematical language. • Teacher checks for the validity of the solution and supporting justification.	• Students report findings should address Formulating, Solving, Interpreting, Evaluating (FSIE).