FORWARD REFLECTION IN TASK DEVELOPMENT AND IMPLEMENTATION

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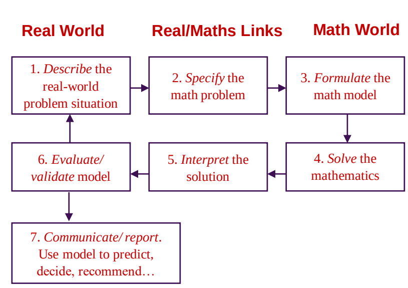
Situation Factors

- A given context can provide a source for several different problems (pre-mathematisation issues).
- A common modelling process (below) can be applied in addressing them.
- But the type and level of mathematics required can differ.
- So can possible role(s) for technology.
- Thinking forward in terms of what may potentially emerge is an important activity in both:

~ task design
and
~ in deciding how to support student activity during implementation.



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Presentation Approach:

Work through problem – following each section, consider forward thinking relevant to the section just completed.

Forward thinking is a practical activity that draws on notions of *implemented anticipation* (Mogens Niss) and *anticipatory metacognition* (Peter G)

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Sample activity: Waste not, want not

Task background

The notion of waste covers a wide spectrum.

It includes household waste for which as individuals we are personally responsible, but also waste generated by industrial processes, and in construction of roads, buildings and so on.

This type of waste is generated on our behalf as members of a population.

This is why the national generation of waste is often expressed in terms of kg (or tonnes) per person.

https://www.abc.net.au/news/2019-12-27/where-does-all-australias-waste-go/11755424

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State of Waste 2016 – current and future Australian trends (MRA consulting)

- On 16 February 2016, the Australian population reached 24 million people.
- Waste generation rates are a function of population growth, the level of urbanisation and per capita income and Australians now produce about 50 million tonnes of waste each year, averaging over 2 tonnes per person.
- There are more of us and we generate more waste per person, each year.
- In the period 1996-2015 our population rose by 28% but waste generation increased by 170%.

Sourced from https://blog.mraconsulting.com.au/2016/04/20/state-of-waste-2016-current-and-future-australian-trends/

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1. Specify real-world problem

Waste statistics are alarming. If the recent rates of increasing waste production continue, estimate the amount of waste that would be generated in Australia over the next century.

2. Specify modelling questions in mathematical terms

If recent rates of waste generation continue:

(a) find a *formula* that predicts the amount of waste that would be produced in Australia in any given future year from 2016

(b) find *estimates*, in *kg (or tonnes)*, for the total amount of waste produced over 25? 50?100? years from 2016.

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Forward reflection

Anticipate, and check features that will be essential for mathematizing a feasible mathematical problem from the real situation

(e.g., if reasonable assumptions together with accessible data are sufficient for formulating a model.)

For this example:

Information about rates of *population growth* and *total waste production*, are provided in the documentation – as are conditions as they apply in early 2016.

These provide direct information sufficient for a basic model and can be disentangled to enable the calculation of waste produced per person as required for a refined model.

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3. Formulate a mathematical model

Assumptions and parameter values

- The MRA figures for Australia of 24 000 000 people producing 50 million tonnes of rubbish at end 2015/beginning 2016 provide suitable initial values.
- Assume population will continue to grow exponentially (compound interest law) – 28% over 20 years.
- Assume growth of total waste follows a similar pattern – thus far 170% over 20 years.
- Assume future average growth rates can be estimated from information about growth over the 20-year period.
- These are sufficient to develop a model to address the specific modelling questions.

We can do it!

Forward reflection

Identify that necessary mathematical techniques are known or are within the scope of the modelling group to find and apply.

For this example:

Recognise that principles of compound interest can be applied to population and waste production, and that hence a feasible approach to the problem exists if these are known.

Realise spreadsheeting can provide an alternative approach which does not require knowledge of geometric series required for the algebraic approach.

This has implications for the level at which the problem might be introduced, and the technology skills required.

4. Solve the mathematics

Compound interest formula:

- After 'n' years compounding growth at r % per year, a quantity with initial value A_0 will have increased to an amount equal to A_n where $A_n = A_0\,(1+r)^{n-1}$.
- In growing by 170 % an initial value of 1 will have grown to a value of 2.7 when n = 20.
- $2.7 = 1(1+w)^{19}$, using 'w' to indicate that we are dealing with the growth of 'waste'.
- Gives w = 0.0537 so that total waste has been growing at 5.37% per year from 1996 to 2015

Take initial value (W_0) of waste produced per year = 50 000 000 tonnes. (end 2015 value).

Assume waste production continues at the compounding rate of w = 0.0537 per year.

So, for question (a) amount of waste produced *in nth year* from 2016 is equal to $W_n = 50000000(1.0537)^{n-1}$

For example:

Waste produced in 2040 (n = 25) = $50000000(1.0537)^{24}$ ≈ 175 459 758 tonnes

Waste produced in 2065(n = 50) = $50000000(1.0537)^{49}$ ≈ 648 786 832 tonnes

Waste produced in 2115 (n = 100) = $50000000(1.0537)^{99}$ ≈ 8 870 559 828 tonnes

Question (b) involves calculating the *total waste* produced over the 25, 50, 100 - year period.

So we need *to add up* the amounts produced in each of the years – the sum of a *geometric series.*

Total waste (T_n) generated over 'n' years from 2016: $T_n = W_0 (w^n - 1)/(w-1)$

W_0 = 50 000 000; w = 0.0537, n = 1…100.

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Using technology (Scientific Calculator) with knowledge of GPs

W_0 =50 000 000; w = 0.0537; n = 25, 50, 100

e.g., $T_{100} = 50000000)(1.0537^{100} - 1)/(1.0537 – 1) \approx 1.73127 \times 10^{11}$ (tonnes)

T_{100} ≈ 173 127 000 000 (tonnes) or 0.17 trillion tonnes (approx.)

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4. Solve the mathematics (alternative)

Using technology (spreadsheet) – knowledge of GPs not needed:

- *Required mathematical knowledge*: amount next year = amount this year + yearly increase
- yearly increase = amount this year*annual rate of increase
- *Required spreadsheet knowledge*: Entering data; coding cell formulae; using copy command

	A	B	C	D
1	*year*	*parameters*	*waste/year (tonne)*	*total waste (tonne)*
2	2016	waste growth fraction (1/year)	50000000	50000000
3	2017	0.0537	52685000	102685000
4	2018		COPY	COPY
5	‘’	waste_init value (tonne)	“	“
6		50000000	“	“
7			“	“
‘			“	“
‘			“	“
101	2115		8870559828	1.73127E+11

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Forward reflection

Think forward about the utility of the selected mathematization, and the resulting output, to provide mathematical solutions to questions posed. (Therefore, anticipate mathematical procedures and strategies to be used after mathematization is complete.)

For this example:

Examine the proposed model to re-check that the mathematization includes all identified factors of importance.

For the basic model these are the initial value of waste/year and an estimate of its average growth rate over time (both accessible).

The formula $W_n = W_0(1+w)^{n-1}$ reflecting compound growth, and the corresponding geometric series sum to represent total waste generated, suffice to deal with the basic modelling problem.

Spreadsheet alternative provides approach not requiring geometric series.

What is needed in the way of technology and mathematical knowledge for both approaches?

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5. Interpret model output

- If there is no change to the rate at which it is generated, then about 0.17 trillion tonnes of waste will be produced in Australia over the next century.
- It is difficult to visualise what this would look like, so it is useful to relate it to something within common experience.
- The Gabba oval in Brisbane has a playing surface area of about 20 085 m^2 (Google search)
- Imagine all waste piled on this surface so as to form a giant cylinder. The height of this cylinder will provide something to relate to.
- First, convert weight to volume. Compacted waste prepared for landfill has an average value of about 750kg/m^3. http://www.pc.gov.au/data/assets/pdf_file/0016/21904/sub028.pdf
- The weight of waste produced over 100 years = 1.731×10^{11} (tonnes) ≈1.731×10^{14} (kg).
- With the density assumption the volume of this waste = $1.731 \times 10^{14}/750 \approx 2.3084 \times 10^{11}\ m^3$.
- Area of Gabba surface = 20 085 m^3. So height of "Gabba cylinder" = $2.3084\times10^{11}/20\ 085$ m ≈ 11 492 km.
- This is within 100 km of the distance from Brisbane to Los Angeles!

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Forward reflection

Think forward to identify needs related to interpretation

For this example:

- Useful to provide a reference base for interpreting large quantities (also for report audience).
- Hence construction of Gabba cylinder.

6. Evaluate the model

First check that the mathematics is correct.

- The model and its calculations reflect the assumptions.
- The numerical data and growth rates reflect the real data – covering the 20-year period from 1996 to 2015.
- In terms of the real world the “Gabba cylinder” height suggests that model predictions are too large to be realistic long term.
- How would a population cope with that amount of rubbish even over time?
- Model looks valid in terms of its assumptions (check sensitivity to small changes in parameter values).
- But is it useful (in real-world terms) for addressing the waste issue?
- The results of the model reinforce the seriousness of the waste problem but led to results unsustainable in the long term.
- No suggestions for addressing the emerging problem are provided by outcomes so far achieved.
- The need for another round of modelling is suggested.

Refinement

A further round of modelling is undertaken, following the same stage progressions.

- Reviewing the MRA consulting data indicates that there are two contributing growth factors at work – population and (waste production/person).
- Total waste production can be modelled as the product of the annual growth rates of *population* (p) and *waste generated per person* (r).
- With initial values as P_0 and R_0 in year 'n' we have, $P_n = P_0 (1 + p)^{n-1}$ and $R_n = R_0 (1+r)^{n-1}$
- *For p*: Population increase = 28% over 20 years so $1.28 = 1(1+p)^{19}$ giving p = 0.0131.
- Population is growing at an approximate rate of 1.31 % per year.
- *For r*: We have: (1+p) (1+r) = (1+w) so that 1.0131 (1+r) = 1.0537.1+ r = 1.0401 (so waste/person is compounding at an approximate rate of 4.01% pa).

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Refinement

- Waste generated in year 'n' is: $W_n = P_n * R_n$
- So, $W_n = P_0 (1+p)^n * R_0(1+r)^n$ noting that $P_0 R_0 = 50\,000\,000$
- Check $W_{100} = 50000000(1.0131)^{99}(1.0401)^{99} \approx 8\,891\,678\,794$ (tonnes/year)
- This compares with the value of 8 870 559 828 tonnes calculated from the original model.
- The values agree to within about 0.24%. (rounding errors)
- Cumulative waste = $P_0 R_0 (t^{100} - 1)/(t - 1)$ where $t = (1+p)(1+r) \approx 1.7434 \times 10^{11}$ tonnes.
- Compares with 1.7313×10^{11} tonne using the previous method. (0.7% difference)
- *A spreadsheet solution can be obtained as for the original model.*

Interpretation

- The first model was purely descriptive in the sense that it showed the future implications of the continuation of a compounding growth in total waste over time - but no basis for action.
- "All of our recycling effort has been taken up by the growth in waste generation (driven by increased per capita consumption and population increases) such that we have made few in-roads on actually reducing waste to landfill."

https://www.insidewaste.com.au/index.php/2019/08/14/a-review-of-the-state-of-waste-in-australia-in-2019/

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- The refined model expresses the mechanism in terms of two factors; population growth, and waste generated per person per year.
- In terms of intervention population growth would be difficult to influence – skilled immigration is required for economic growth.
- Could we make an impact by reducing the amount of waste generated per person, for example through recycling, more efficient industrial processes, and increased personal responsibility?
- Check: A 10% reduction in the average growth rate of waste/person/year from 4.01% to 3.61% reduces the total waste produced over100 years by more than 25%.
- For a 50% reduction the reduced figure is more than 77%.
- Modelling has given insight into where efforts should be directed (or continued) to make a substantive difference to the future waste problem – in reducing the per capita production of waste, and in increasing the efficiency of recycling that which is generated.

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Forward reflection

Thinking forward to identify further problems that are suggested by progress so far made. Some of these may not have been thought of at the outset of the original problem.

For this example:

- Realisation that solution to basic problem, while usefully descriptive, does not provide leverage for action.
- Evaluation in terms of real-world significance leads to the search for a more useful (refined) model.
- Expressing total waste as the product of *population* (persons) and *waste generated per person* enables the impacts of population growth and of growth of waste/person to be treated separately.
- There is now leverage in exploring how methods such as improved recycling would have a predicted impact on the fundamental problem.
- Early reflection might identify that the refinement model would be the most appropriate formulation from the start?

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6. Report the outcomes

A report would consist of a smooth synthesis of the material in sections 1 to 6, in which the original model and its refinement would feature.

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