THE ENABLING OF MATHEMATICAL MODELLING IN SECONDARY SCHOOLS ZOOM SYMPOSIUM 2021: Introduction

PROFESSOR VINCE GEIGER AND
PROFESSOR PETER GALBRAITH

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Researchers

Vince Geiger (Institute of Learning Sciences and Teacher Education, Australian Catholic University)

Peter Galbraith (The University of Queensland)

Mogens Niss (Roskilde University, Denmark)

Gloria Stillman (Australian Catholic University)

Jill Brown (Deakin University)

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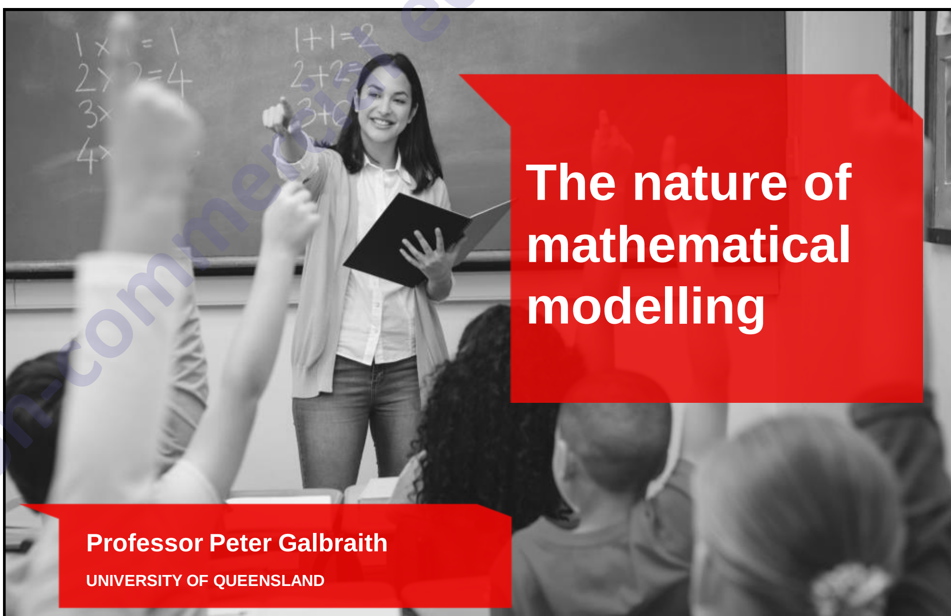
Using mathematics to solve real-world problems: The role of enablers

The purpose of this project is to identify, apply and refine teaching approaches that help secondary students learn how to use mathematics to solve real world problems; the processes of mathematical modelling.

The study is investigating the factors that "enable" the modelling process including the design of tasks that support students' development as modellers, and effective teaching approaches that promote student capability and interest in mathematics



The nature of mathematical modelling

Professor Peter Galbraith
UNIVERSITY OF QUEENSLAND

THE UNIVERSITY OF QUEENSLAND AUSTRALIA

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Mathematical Modelling as Real-World Problem Solving

"Mathematics aims to ensure that students are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their *personal* and *work* lives and as *active citizens*."

Australian Curriculum Assessment and Reporting Authority (2017).

Similar statements from OECD, USA, Singapore…

- This means developing in students the ability to be real-world problem solvers.
- Mathematical modelling is the (only!) place in the curriculum where this purpose can be seriously addressed.
- Spending up to 12 years studying mathematics and being able to use acquired knowledge only for examination purposes or doing text-book exercises is unacceptable.
- Nothing is more emancipatory than enabling students to use whatever level of mathematics they have learned to address problems in their world.
- Evidence indicates that modelling can be successfully taught, learned, and applied independently.

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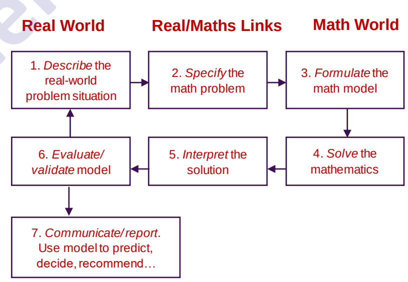
A. Modelling Process



- Describes a generic problem-solving process applicable to addressing any modelling problem (various diagrammatic depictions exist.)
- The arrows indicate the ordered logic of the modelling process e.g., mathematisation (formulation) cannot occur until a mathematical question has been decided; interpretation requires that a mathematical outcome has been obtained and so on.
- The box labels can be useful as headings when structuring a report.

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B. Modelling Routes

- These are the actual solution paths taken by modellers.
- They typically vary markedly, featuring back tracking between and within stages, as consequences of reviews of progress, making corrections, revising assumptions and parameter values etc.
- Their detail is relevant for providing individual or group assistance during implementation (measured responsiveness).

Mathematical Modelling as Real-World Problem Solving

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C. Report Writing

- Report writing involves producing and evaluating a problem solution – written in terms of the modelling process sequence.
- Reports should be written for an intelligent lay audience who can understand mathematics that is explained to them. They cannot be expected to fill gaps or read minds.
- Details of specific meanderings within modelling routes are not material for reports.

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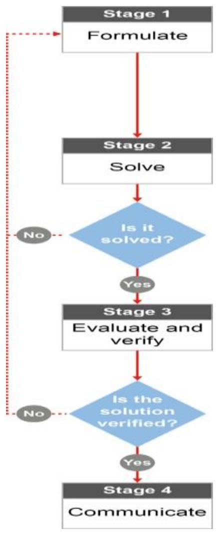
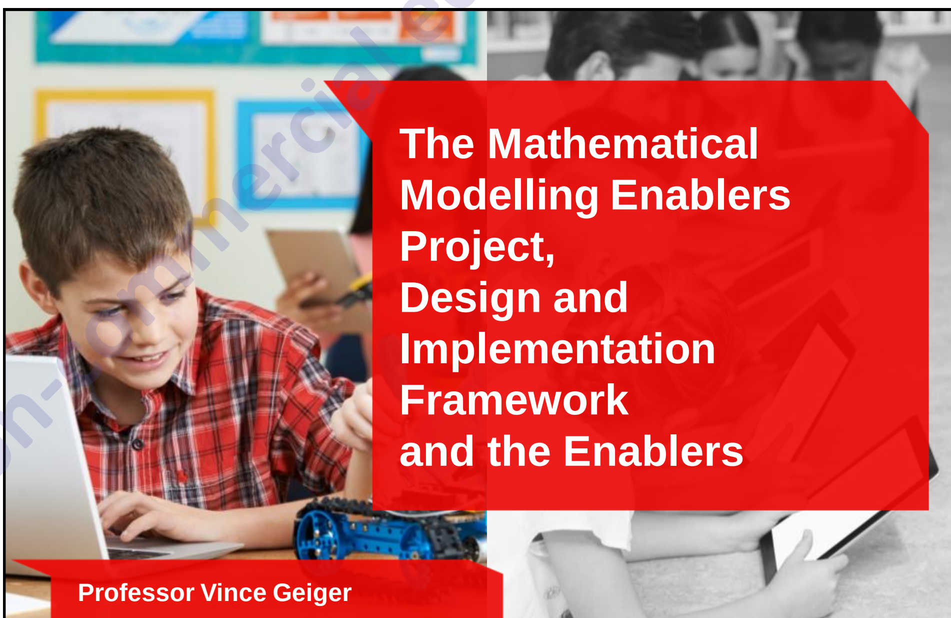
Syllabus compatibility

From QCAA: Senior syllabuses extract (Stage 0 added); Stage 0 (Problem Statement)



- Through developing and applying mathematical models, students cumulatively become real-world problem-solvers.
- Ultimately, this means that not only can they productively address problems set by others, but also that they develop the ability to identify and address problems and answer questions that matter to them.
- Modelling needs to start in Junior School



The Mathematical Modelling Enablers Project, Design and Implementation Framework and the Enablers

Professor Vince Geiger

INSTITUTE OF LEARNING SCIENCES AND TEACHER EDUCATION, ACU

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Project context

- Secondary students between Years 9-11.
- Curriculum context – new assessment system; modelling/problem solving included; 20% of assessment
- Teachers from different schools followed over three years.
- Three whole day teacher/researcher meetings per year. Video-recorded classroom observations and teacher and student interviews during school visits between each meeting. Small groups of students participate in video-stimulated recall sessions after each school visit.
- Tasks initially developed by researchers with teachers becoming increasingly involved in their design.
- Iterative development of a task design and implementation framework for in collaboration with teachers.

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Anticipation and modelling

A successful modeller is able to *anticipate*, and to project her/himself into, subsequent modelling steps before actually taking them. Such anticipation is essential throughout the modelling process.

(Niss, 2010; Niss (Martin), 2017; Jankvist & Niss, 2019).

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Anticipation and modelling

- **Anticipating features that are essential** in mathematising a feasible problem from the real situation.
- **Anticipating mathematical representations and mathematical questions** that, from previous experience, or present analysis, seem likely to be effective when forming a mathematical model
- **Thinking forward about the utility** of the selected mathematisation, and the resulting model, to provide a mathematical solution to the questions posed. (Therefore, anticipating mathematical procedures and strategies to be used in problem solving after mathematisation is complete.)
- **Thinking forward to identify related problems and refinements** that are suggested by progress so far made. Some of these may not have been thought of at the outset of the problem.

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Enablers of implemented anticipation

ME1
Teachers and students believe that the inclusion of modelling activities is a valid component of mathematical coursework and assessment

ME2
Students possess mathematical knowledge able to support modelling activities (e.g., possess mathematical knowledge and skills, and ability to manage abstraction)

ME3
Students possess an understanding of a systematic modelling process that includes successive stages from problem question to model evaluation

ME4
Students are capable of using their mathematical knowledge when modelling (Implies a core understanding of and engagement with the modelling process (Formulate, Solve, Interpret, Evaluate) so that the right questions can be asked and pursued systematically)

ME5
Students have perseverance and confidence in their mathematical capabilities (e.g., continue to follow through, or try new directions within a problem)

(adapted for Australia from Niss, 2010)

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Enablers (implementation)

Core teaching enabler
- Teacher enabling the student to utilise modelling process (Teacher/Student)

Learning/Teaching Environment
- Classroom/school culture which supports investigative approaches (socio-mathematical norms – didactical contract)
- Alignment with learning goals

Teaching enablers
- Teachers' anticipatory sets
- Measured responsiveness (principle of minimal support – Aebli; wild good chasing – Schoenfeld)

Catalytic enablers
- Collaborative engagement with peers (teacher/pupil; debate and resolution; bootstrapping)
- Use of technology
- Student interest in the problem (more about students' beliefs about applications than interest – mathematical beliefs) Useful in the sense of what they might need somewhere else.

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How can these enablers be implemented in classrooms?

Task design and implementation are two sides of the same coin – both require anticipation.

Design and Implementation Framework for Mathematical Modelling Tasks (DIFMT)

Principles for mathematical modelling task design – Design

Principle 1:	There is some genuine link with the real world of the students.
Principle 2:	There is opportunity to identify and specify mathematically tractable questions from a general problem statement.
Principle 3:	Formulation of a solution process is feasible, involving the use of mathematics available to students, the making of necessary assumptions, and the assembly of necessary data.
Principle 4:	Solution of the mathematics for the basic problem is possible for the students, together with interpretation.
Principle 5:	An evaluation procedure is available that enables checking for mathematical accuracy, and for the appropriateness of the solution with respect to the contextual setting.
Didactical principle:	The problem may be structured into sequential questions that retain the integrity of the real situation. (These may be given as occasional hints or used to provide organized assistance by scaffolding a line of investigation.)

Pedagogical architecture - Implementation

Pre-engagement - Understanding of the modelling process and its application including support materials (learn/illustrate what the modelling process is)

Modelling process review – Reviewing pre-engagement as required.

Pedagogical architecture - Implementation

Initial problem presentation

- Teacher provides brief general description of the problem scenario (sets the scene) [2-3 minutes]
- Students should be organised into small groups. They are provided with time to read the task description and ask questions of clarification.
- In small groups students discuss how to approach the problem (e.g., What is the mathematical question?) and report back to whole class via a group representative.
- Teacher should orchestrate discussion of the mathematical question towards consensus.
- Students in groups then consider assumptions and variables relevant to the mathematical question as well as other observations such as trends in data, dimensional analysis, etc. Outcomes should be reported back to whole class by a group representative.
- Teacher synthesises/prioritises students' initial assumptions and variables sufficient to begin modelling process for an initial model.

Design and Implementation Framework

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Pedagogical architecture - Implementation

Body of Lesson – Students:

- Proceed in their groups to create model, solve, interpret, etc in terms of the question they are addressing.
- Engage in productive student-student collaboration
- Identify and make productive use of technology where applicable, for example, to source relevant information, check calculations and/or generate solutions.
- Develop a report of their progress in terms of the stages of the modelling process (e.g., formulate, solve, interpret, evaluate)

Pedagogical architecture - Implementation

Body of Lesson – Teachers:

- Bring to consciousness those things that are implicit …actions are then deliberate.
- Activate teacher meta-meta cognition: (a) How will the students be interpreting what I as a teacher am doing/saying at this point? (b) What should the students be asking themselves at this point in the modelling process?
- Anticipate where students might have problems, e.g., interpreting the problem, generalizing the solution.
- Employ measured responsiveness – rather than providing specific advice about the problem, teachers should prompt students to think about where they are in the modelling process. Structure mathematical questions that promote a viable solution pathway.
- Encourage the use of digital or other tools as appropriate.
- Support student development of a modelling report.

Conclusion

Presentation of findings and teacher summary

- Students share what they have found with justification (representative from each group as spokesperson). Findings should be reported in a succinct fashion (e.g., via 3-4 minute video)
- Teachers/students ask questions of clarification as required or to test arguments.

Report

- Students should communicate their findings via a succinct, coherent, systematic report. The report must make use of appropriate mathematical language.
- Teacher checks for the validity of the solution and supporting justification.

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Fundraising for schools, fetes, and clubs often involves running stalls at which items are sold at a price that makes the effort and prior expenses worthwhile. To achieve this, event organisers need to determine how much to charge per item in order to achieve a profit.

Your school intends to organise a sausage sizzle.

What price should be charged per sausage sandwich?

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Specifying a problem

What price should be charged per sausage sandwich if sales should produce a small profit?

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Specifying a mathematical question

What should be the price per sausage sandwich (c) if the sausage sizzle is to turn a profit (p)?

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Formulate a mathematical model: Key sub-questions, assumptions and variables

To determine the cost of a sausage sandwich (c).

- What is the target profit (p)? – Assume $500 (Assumption needs to be justified, e.g., what has been the typical profit from previous years)
- How many people are likely to buy a sausage sandwich (n) – Assume 300 (Assumption needs to be justified, e.g., typical numbers that attend the fate, percent vegetarians [assume similar to Australian population], percentage who will choose a sausage sandwich to eat, etc.)
- How much do each of the following items cost: slice of bread (b); a sausage (s); tablespoon of onions (o); squeeze of tomato/BBQ sauce (t); gas for BBQ (g); oil for BBQ (oil).

(Each to be justified, e.g., use of internet to find price of items online. Some costs determined by investigation, e.g., cost of a tablespoon of onion. Some assumptions might require polling – how many people would include onion)

(broad assumptions – volunteer labour, school owed BBQ at no cost)

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Body of lesson

- Students create model, solve, interpret, etc in terms of the question they are addressing (optimally in groups).
- Anticipate where students might have problems, e.g., interpreting the problem, identifying assumptions and variables.
- Employ measured responsiveness –teachers should prompt students to think about where they are in the modelling process and not offer advice about specific approaches to solution.
- Multiple solution pathways should be encouraged.
- Encourage the use of digital or other tools as appropriate, for example, spreadsheets.
- Responses to students' questions or requests for assistance could include: What are you doing? What are you trying to do? Where are you in the modelling process? How have you checked your answer? (both mathematically and in terms of context)?

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Formulate a mathematical model

Initial model (assuming everyone has onion)

\$500 = 300 x c – 300 x (s + o + b + t) – (g + oil)

Variables **s**, **o**, **b**, **t**, **g** and **oil** to be substituted with values based on justified assumptions or the outcomes of investigations.

Model solved for c for an estimate of the cost of a sausage sandwich.

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Evaluate

- Check all calculations are correct.
- Does the solution make sense against the original context (e.g., estimate what might be sensible answers…$2?...$20).
- Is the outcome affordable in the context of a school fete?
- Are there ways of checking this is the best solution (e.g., try different inputs for price of sausage, bread…)
- Check your solution (assumptions and variables) against those of others.

Example Modelling Task

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Refinement

- What other assumptions might be made?
- What other variables could be incorporated?
- Can estimates of inputs be improved?
- Could a variety of sausage sandwiches be offered at the stall (e.g., hot dogs, gourmet)

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Discussion/Summary

It is important that:

- both students and teachers are familiar with the modelling process.
- students see working with applications of mathematics as important even when they have a preference for working with pure mathematics.
- students clarify the mathematical question before developing assumptions.
- teachers developed their anticipatory sets (must work through problems carefully themselves).
- socio-mathematical norms and/or the didactical contract are understood by teachers and students.
- teachers' work with modelling complements their perceptions of curriculum expectations.
- collaboration is seen by both teachers and students as important to the development of modelling capability – but how does this play out in assessment contexts?

The University Of Queensland Australia

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