Developing the teaching and learning of modelling – an international perspective

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Overview of presentation

Introduction and historical outline

Recent advances and focal points

- **Setting the stage**
- **Descriptive and prescriptive mathematical modelling**
- **Modelling in mathematics education**
- **Modelling competency and (sub-)competencies**
- **Modelling difficulties**

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The formative years, 1970-1990: Foci

Applications, models and modelling on the agenda (in certain countries) since the 1970s. Pioneers include UK, Australia, Germany, Denmark, the Netherlands. More countries ”joined” later.

The **ICTMA**s (International Conference on the Teaching of Mathematical and Applications), forum since 1983 (Exeter, UK)

Pleas for inclusion of A(pplications) & M(odelling) in the teaching of maths, based on positive show cases

Putting forward reasons and arguments

Need for conceptual clarification (application, model, modelling)

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Proposing and discussing ways to introduce A & M in mathematics curricula – integrated (if so, how?) or separate (if so, how?).

Presenting cases of experimental teaching and teaching materials.

Teaching and learning primarily concerned students' working on existing applications and models, rather than on constructing new ones.

Academic work was predominantly of a **developmental** nature.

Research proper was focused on conceptual clarification and theoretical constructs; empirical research was scarce in those years.

The years of consolidation: 1990-2010: Foci

During the 1990s, emphasis changed towards students' construction of models, i.e. the processes of **modelling**

Modelling is difficult and demanding

Mathematical knowledge and skills are **not sufficient**!

What more is needed?

Modelling can be learnt and cultivated, but requires sustained effort and investment of immaterial and material resources! **Calls for flexibility!**

This certainly comes at a cost!

Key task: Designing teaching/learning environments that enable students to undertake modelling work.

This gave rise to **theoretical and empirical research on modelling** in mathematics education, concerning:

- The nature of the modelling process – both theoretically and empirically
- Characterising modelling competency and capabilities
- Identifying the prevalence of modelling competency with different categories of students
- Assessing students' modelling achievements with respect to different approaches to the teaching of modelling.

Recent advances and focal points of r & d

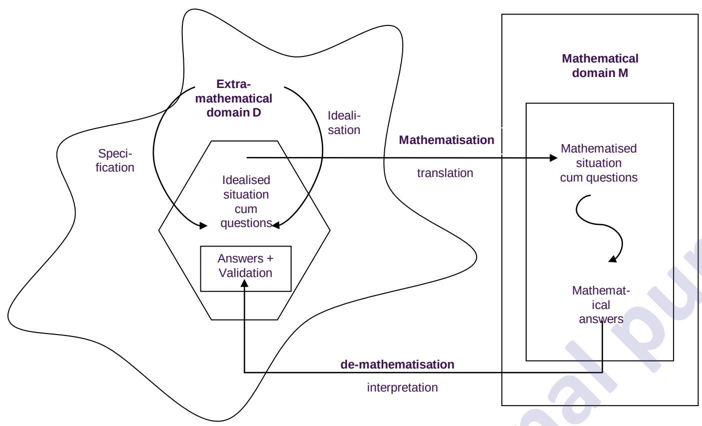
Setting the stage: basic notions and terms

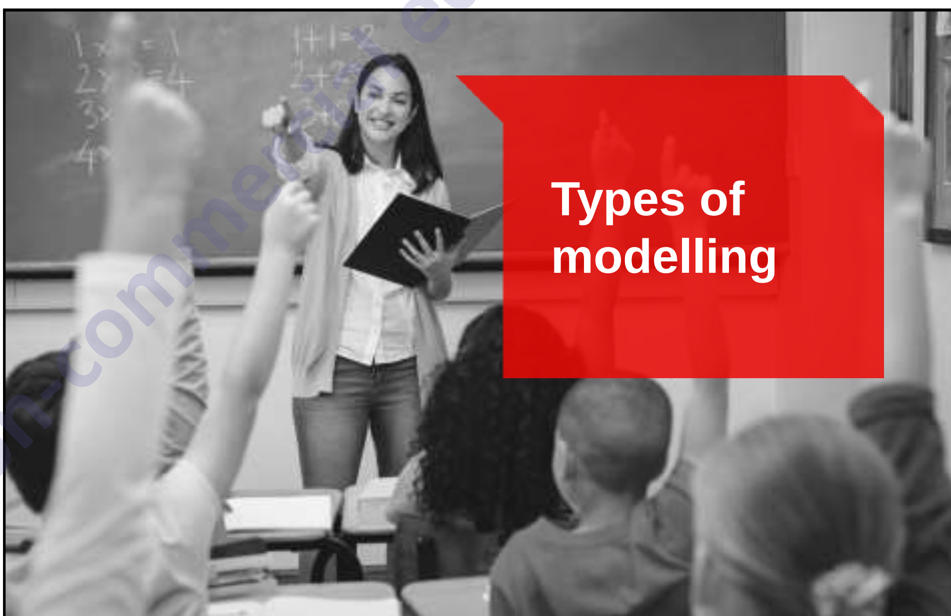
Classically:

Modelling is undertaken to **answer questions** concerning a context and situation in some extra-mathematical domain, in order to, e.g.,

- understand the situation
- test hypotheses
- solve extra-mathematical problems
- explain phenomena
- predict future events
- pave the way for decision-making

The modelling cycle





Types of modelling

So far, we have focused on **capturing an existing reality** by way of modelling. We have undertaken **descriptive modelling**.

However, there is also another kind of modelling: **Prescriptive modelling**, in which we attempt to structure, design, organise and construct reality, i.e., we are **creating reality**.

Examples: pH value; velocity (including speed) and acceleration; density; growth rate; the BMI Index; the Gini coefficient; design of physical objects to meet certain requirements; design of social artefacts, e.g. annuity loans, pension schemes, insurance premiums, allocation of resources, location of plants or institutions; devising voting systems and defining election outcomes.

Note: We distinguish between descriptive and prescriptive modelling, **not** between descriptive and prescriptive models.

The **modelling cycle can be used to capture prescriptive modelling, too,** except w.r.t. validation and evaluation.

In the context of prescriptive modelling, validation of model outcomes, and evaluation of models **differ** from the situation in descriptive modelling.

Since prescriptive modelling is meant to create or shape reality, it **doesn't make sense** to speak of a "correct model" based on confrontation of the model outcomes with reality before it has been created or shaped.

Instead, **focus on** sensibility, relevance, appropriateness, expediency, usefulness.

The focus on prescriptive modelling is of a **recent date**, (Niss, 2015) even though the term was introduced already by Phil. J. Davis (1991).

Two distinct purposes – non-contradicting, but with different consequences:

Modelling as a goal in itself in mathematics education

- Doesn't result automatically from mathematical knowledge and skills
- Modelling has to be – and can be – learnt. However, this has non-trivial consequences in terms of investment of time, resources and activities, extra-mathematical work, i.e. **in terms of flexibility of teaching and learning**

Modelling as a vehicle for something else, above all the learning of mathematics

- Motivation, sense-making, underpinning and consolidation of concepts, methods and results
- Many mathematical concepts originated as modelling concepts from prescriptive modelling (e.g., proportionality, ratio, derivative)

Modelling competency and (sub-)competencies

As modelling is difficult and demanding there is a need to define and characterise modelling competency.

Two different conceptualisations:

- **Top-down** conceptualisation: Define a comprehensive, overarching modelling competency and look for its constituents.
- **Bottom-up** conceptualisation: Identify a number of separate modelling competencies and then bundle them together.

Top-down – or comprehensive – conceptualisation, primarily taken by, for example, Danish and Australian researchers, including our ARC project!:

The ability to undertake *active model construction as a whole* in given extra-mathematical contexts, as well as the ability to analyse and assess extant mathematical models.

The bottom-up – or piecemeal – conceptualisation, primarily taken by, for example, German researchers:

Understand the extra-mathematical problem and set up a real model; Set up a mathematical model from the real model; Solve mathematical problems within the mathematical model; Interpret mathematical results in the real situation; Validate the solution.

Fostering and furthering students' ability to model must focus on devising ways to **develop their modelling competency and associated (sub-) competencies**.

But how? Holistically – focusing on tasks that involve the full modelling cycle? Or **atomistically** – focusing on different tasks, each involving one or a very few modelling stages only?

Combining the two approaches - but not at the same time, of course – seems to be desirable!

Modelling difficulties

International research has identified lots of **stumbling blocks** (obstacles or blockages) to students' learning of modelling.

What are they, and how can they be counteracted?

At first, stumbling blocks were identified **in most of the transition stages** in the modelling cycle: Mathematisation; Mathematical problem solving; De-mathematisation; Validation/evaluation; Plus making assumptions in pre-mathematisation.

Mathematisation and mathematical **problem solving** have been shown to constitute significant stumbling blocks.

Recent research has found many stumbling blocks in the **pre-mathematisation** stage:

- Tailoring and idealising the situation to be modelled
- Selecting the essential entities to be considered and discarding the less essential ones
- Making simplifying but not too simplistic assumptions,
- Finding information and data to underpin the modelling process

are all much harder than one would think. They **all constitute potential stumbling blocks**. Sometimes they can even prevent the modelling work from getting started.

An overarching stumbling block, **common to several stages of the modelling cycle**, (e.g. pre-mathematisation, mathematisation, mathematical problem solving, and validation of outcomes) is the need for the modeller to anticipate what can/must be done in subsequent stages, and to **implement** that anticipation in advance.

Implemented anticipation

i.e. implementing the anticipation of future steps *now*, before the steps have been made!

Initially **a theoretical construct** (Niss, 2010), later corroborated by **empirical findings** (Brown & Stillman; Niss; Niss Ma., Niss & Jankvist, Geiger, Galbraith & Niss).

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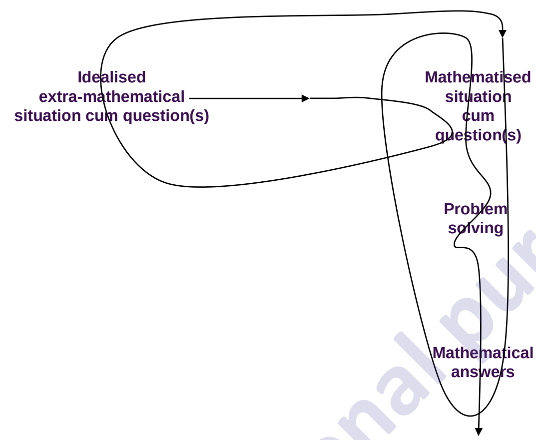
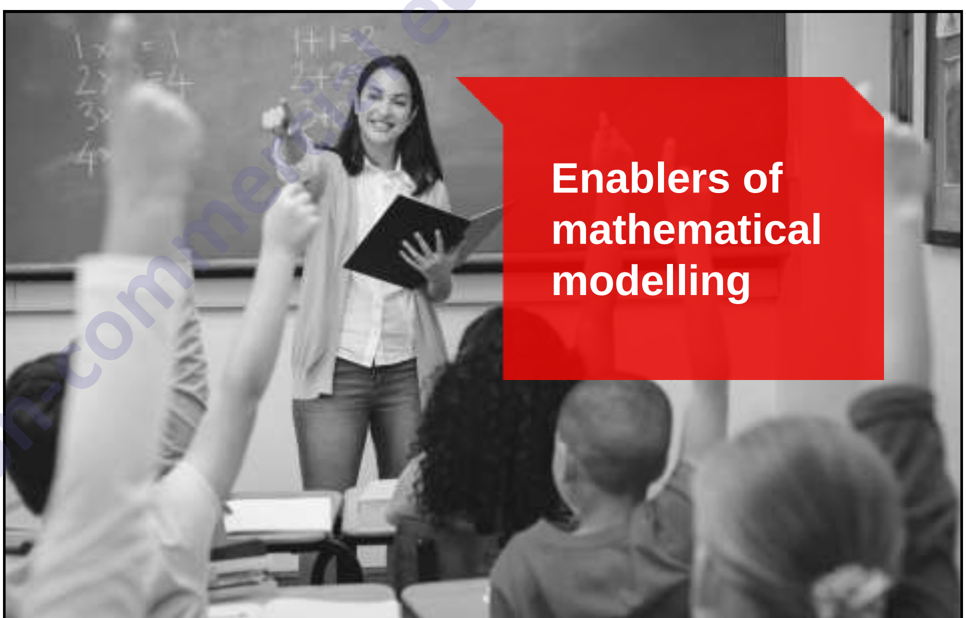
The itinerary of implemented anticipation





Enablers of mathematical modelling

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There are several **different enablers** of mathematical modelling – and **disenablers** as well!

Key challenge: **To enable students to undertake implemented anticipation.**

For students to become bold “implemented anticipators” they must experience working on a wide spectrum of small and large scale, authentic modelling situations, involving both frustration and success. Intravenous injections don’t suffice.

Requires that the teaching-learning environment allows for the such experiences to be developed with students.

Creating such an environment may well require a **change/renegotiation of**

- The didactic contract (Brousseau)
- The socio-mathematical norms in the classroom (Cobb & Yackel)
- Students’ (and teachers’?) mathematical beliefs (Schoenfeld, Törner, Jankvist)
- Formative and summative assessment:

    WYAWYG: What you assess is what you get!

But, if you want to pursue a goal you must accept the means to reach it.

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Concluding remarks

The more important mathematical modelling becomes in society and mathematics education, the **more important it is to foster and further it with students**.

Modelling is **difficult and demanding but can be learnt**. **But at a cost**!

This requires deeper insight into the nature of the modelling process, the challenges this poses to learners, and into ways to overcome them. Fortunately, **we already know a lot** about this.

So, research is alive and kicking, but we need much more of it!

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