DESIGNING A MODELLING TASK

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Plan for this session…

1. Why modelling?
2. Examples/ideas for modelling tasks.
3. Try to create a modelling task.
4. Sources of ideas for tasks.
5. Sometimes they do not go as expected.
6. Questions

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Australian Syllabus Review

acara

Australian Curriculum Review

Mathematical modelling

The Australian Curriculum: Mathematics recognises the importance of mathematical modelling to the development of conceptual understanding and application of mathematical structures. Modelling is central to the contemporary discipline of mathematics and is fundamental to the practical application of mathematics. Mathematical modelling is the process of using mathematics to make decisions, predict outcomes and understand relationships that exist in authentic real-world scenarios by mathematising a situation, recognising, connecting and applying mathematical structures and using mathematical approaches to manipulate, analyse, generalise, interpret and communicate within the context of the modelling situation. A key aspect of modelling is to identify and attend to key aspects of a situation or context while ignoring others. This enables a simpler version to be constructed for a particular purpose and predictions to be made based on the model, which can then be tested and the model validated or further refined.

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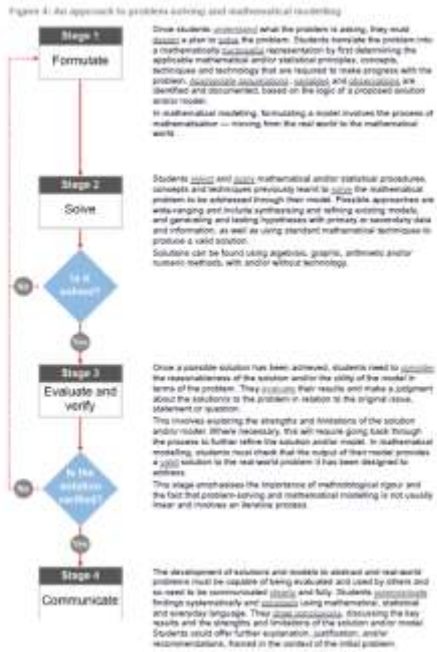
QCAA Senior Syllabuses

Problem-solving and mathematical modelling

A key aspect of learning mathematics is to develop strategic competence; that is, to formulate, represent and solve mathematical problems (Kilpatrick, Swafford & Bradford 2001). As such, problem-solving is a focus of mathematics education research, curriculum and teaching (Sullivan 2011). This focus is not to the exclusion of routine exercises, which are necessary for practising, attaining mastery and being able to respond automatically. But mathematics education in the 21st century goes beyond this to include innovative problems that are complex, unfamiliar and non-routine (Mevarech & Kramarski 2014).

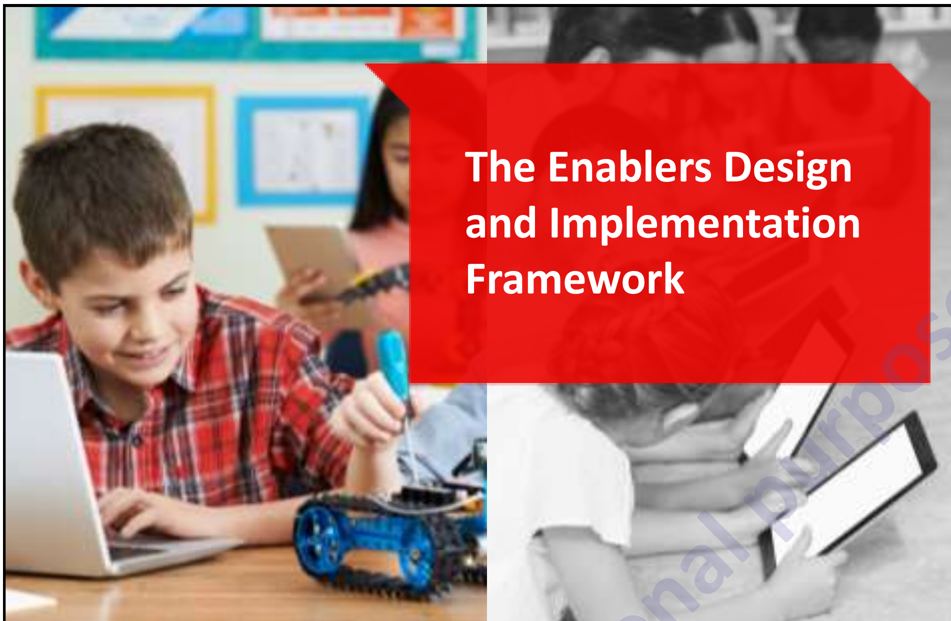
Problem-solving in mathematics can be set in purely mathematical contexts or real-world contexts. When set in the real world, problem-solving in mathematics involves mathematical modelling.



21st century skills

The 21st century skills identified in the following table reflect a common agreement, both in Australia and internationally, on the skills and attributes students need to prepare them for higher education, work and engagement in a complex and rapidly changing world.

21st century skills	Associated skills	21st century skills	Associated skills
critical thinking	• analytical thinking • problem-solving • decision-making • reasoning • reflecting and evaluating • intellectual flexibility	creative thinking	• innovation • initiative and enterprise • curiosity and imagination • creativity • generating and applying new ideas • identifying alternatives • seeing or making new links
communication	• effective oral and written communication • using language, symbols and texts • communicating ideas effectively with diverse audiences	collaboration and teamwork	• relating to others (interacting with others) • recognising and using diverse perspectives • participating and contributing • community connections
personal and social skills	• adaptability/flexibility • management (self, career, time, planning and organising) • character (resilience, mindfulness, open- and fair-mindedness, self-awareness) • leadership • citizenship • cultural awareness • ethical (and moral) understanding	information & communication technologies (ICT) skills	• operations and concepts • accessing and analysing information • being productive users of technology • digital citizenship (being safe, positive and responsible online)



The Enablers Design and Implementation Framework

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Foundation principles

- The modelling process, including a graphical representation, should be understood.
- The modelling process should be applied to a problem before it is presented to students- to be equipped to anticipate the many contingencies
- Report documentation should develop in parallel with progress through the stages of the modelling process.

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Principles for modelling task design

1. Nature of the problem:

Problems must be open-ended and involve both intra- and extra mathematical information.

The degree of open-endedness is dependent on students' previous experience with modelling.

- Less experienced students need additional scaffolding, questions or information.
- Most experienced students should be expected to engage with less defined problems.

Design and Implementation Framework

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Principles for modelling task design

2. Relevance and motivation

- There is some genuine link with the real-world problem of the students.
- This will depend on factors including students' age, year level, personal circumstances etc.
- Problems need to be tractable from the perspective of the student group.

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Principles for modelling task design

3. Accessibility

Is it possible to identify and specify mathematically tractable questions from a general problem statement.

Is there a mathematical approach accessible to students?

Problems must be tractable from the perspective of the student group.

Design and Implementation Framework

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Principles for modelling task design

4. Feasibility of approach

Formulation of a solution process is feasible, involving

a) The use of mathematics available to students;
b) The making of necessary assumptions;
c) The assembly of necessary data.

Teachers must work the problem

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Principles for modelling task design

5. Feasibility of outcome

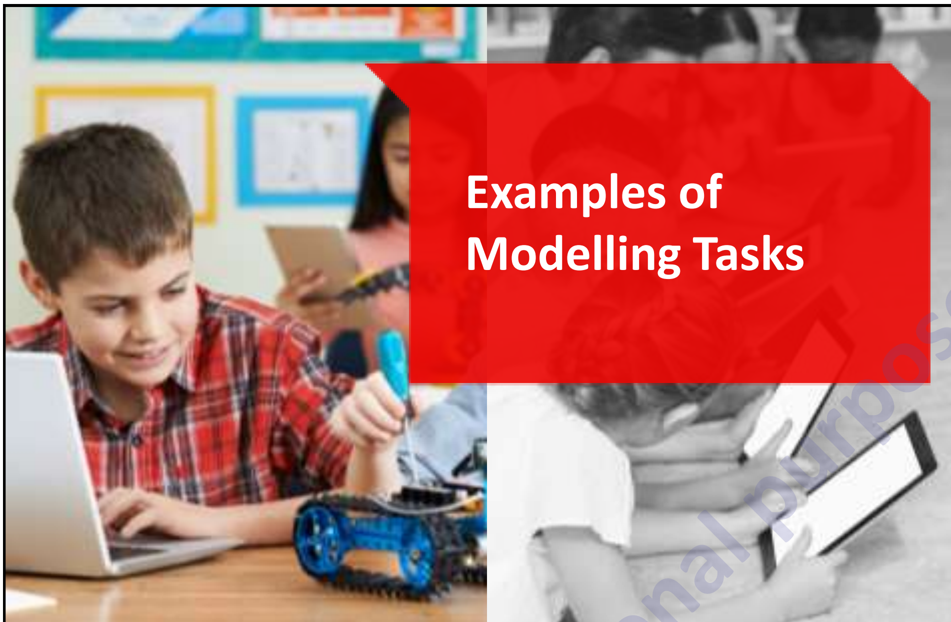
- Solution of the mathematics for a basic problem is possible for the students, together with interpretation.
- Expectations in relation to the type of response, for example arithmetical vs generalised solutions, are dependent on the characteristics and year level of the specific student group.

Principles for modelling task design

6. Didactical flexibility

- The problem may be structured into sequential questions that retain the integrity of the real situation. (Having worked through the problem, how can it be implemented?)

  For example, can prompts/assistance to students be structured into sequential questions (identify the subsections of the problem)?



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Straight-sided Circles

In construction of large circular shapes, engineers and builders often use straight sides as they are easier and cheaper to construct. However, visually it appears the object is circular.

Figure 1 Fish Cages: Diameter 30 metres

Picture source: *https://www.feedstrategy.com/aquaculture/proposed-cooke-aquaculture-farm-opposed-by-local-groups/*

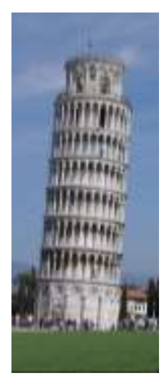

Figure 2 Tower of Pisa: Picture source: Tahbepet CC BY-SA 2.0

Task:

Determine the length of the longest straight side for either the fish cage or the tower of Pisa could have.

Generate a model to determine the length of the longest straight section that can be constructed for a circle of appropriate radius so that it will appear to form the circumference of the circle.

Use the Problem-solving and modelling approach to assist you.

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Straight-sided Circles: Solution

You will probably find that some students will attempt an approach that you have not thought about.

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Boy in a Ball

At a shopping centre recently, I saw this activity.

How long should a person be allowed to stay in the ball?

Dripping tap

Sources claim that a dripping tap will waste 20 000L of water in a year. (An average backyard swimming pool is about 40 000L.)

Build a mathematical model for the loss of water from a dripping tap and use this model to determine whether either of these results are reasonable.

List the assumptions necessary for your model to be valid.

Justify your conclusions mathematically.

Change one of the assumptions and investigate the effect on the result.

Big Iceberg

Show a possible iceberg on the picture below. You need to justify your conclusion.

Use the resources on the next slide to help you.

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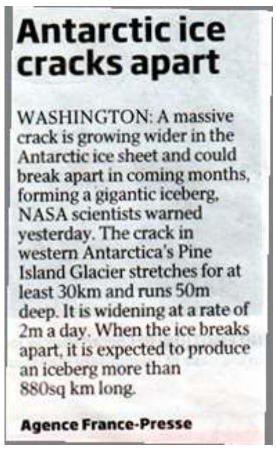
Big Iceberg (continued)

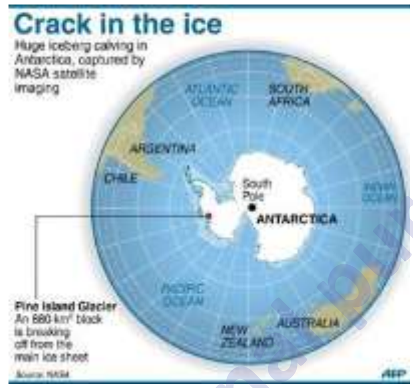
There is an incorrect statement in the article. What is it and why is it incorrect?)



Antarctic ice cracks apart

WASHINGTON: A massive crack is growing wider in the Antarctic ice sheet and could break apart in coming months, forming a gigantic iceberg, NASA scientists warned yesterday. The crack in western Antarctica's Pine Island Glacier stretches for at least 30km and runs 50m deep. It is widening at a rate of 2m a day. When the ice breaks apart, it is expected to produce an iceberg more than 880sq km long.

Agence France-Presse



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Where should we sit at the movies?

- When we go to the movies, one the questions that is always: “Where do you want to sit?"
- Consider a movie theatre with a sloping floor.
- What attributes should the best position have?
- Using these attributes generate a mathematical model and use it to determine the best location to view the movie.

A plate of large proportions

Quite often in archaeologically digs, pieces of pottery are discovered. Archaeologists want to know the size of the plates. The photo has a scale of 1:20.

- Synthesise (build) a strategy and implement it to determine the size of the plate, of which this is a piece.
- Test your strategy on a plate of known size to verify that your strategy works before you use it on this specific example.
- Detail any assumptions you feel necessary for your strategy to be valid.
- Detail any limitations that may cause you to have concerns about the conclusions your strategy leads you to.

Design a dipstick

A dip stick is a long stick with graduations on it and is used to indicate how much fuel is remaining in a tank. The dipstick is placed in the tank and when withdrawn, the reader is able to identify where the dipstick ceases being wet and obtains an indication of the remaining fuel in the tank.

Build a dipstick for the following fuel tank.
The fuel tank is 5m long and has a diameter of 2m.
Detail any necessary assumptions you need to make.
Provide a drawing/table of your dipstick with the amount of fuel in the tank.

Let’s try to build a task

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Building a task

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Idea for a task…

Macca’s ad

- https://youtu.be/xN7FlEebH5g
- Mowing in a circle

Suggestions for a task

Year level

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Some (possibly useful) websites…

- https://my.nctm.org/blogs/sarah-hampton/2019/02/11/examples-of-effective-modeling-activities
- https://serc.carleton.edu/ANGLE/educational_materials/activities/205743.html
- https://serc.carleton.edu/integrate/teaching_materials/energy_and_processes/activity_4.html
- https://serc.carleton.edu/integrate/teaching_materials/energy_and_processes/activity_2.html
- https://serc.carleton.edu/sp/cause/mea/examples.html

Data sites

- https://data.worldbank.org/
- https://data.worldbank.org/indicator/EG.ELC.RNWX.ZS

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Just because you can get it to work, does not mean they will get it to work…

Building paper helicopters

Task:
Design and build a paper helicopter that will reach the ground (3 stories) in exactly 20 seconds.

Experiment with the size of the helicopters. Change their wing sizes and pitch. Drop the helicopter from a height (third story) and record the time it takes to reach the ground.
Investigate what happens if you create a helicopter with:

- Big wings.
- Small wings.

How does changing the pitch effect your helicopters?

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